SECOND YEAR ALGEBRA WORKLOAD

GENERAL DIRECTIONS: Read and study the lessons 1.1 and 1.2.

I. Handout on 1.1 Addition and Subtraction of Polynomials

In size1 with complete solution, legible handwriting and boxed final answers,

a. answer the odd numbers of the 'Need More challenge' part; and
b. for 'Conquer These', answer the even numbers of test A and answer test B.

It is required that you submit this on July 2, 2009.

II. Read the handout 1.2 on the Laws of Exponents.

Lesson 1.1	**Review Addition and Subtraction of Polynomials**
	OBJECTIVE: 1. To distinguish polynomials from other algebraic expressions. 2. To identify the number of terms and the parts of a term. 3. To simplify polynomials. 4. To add and subtract polynomials.

Mathematicians used expressions to model real-world situations. The expression $-16t^2 + 96t$ is a mathematical model of the height (in feet) after t seconds for a projectile with an initial vertical velocity of 96 feet per second.

The expression $-16t^2 + 96t$ is a polynomial.

Polynomial is of the form $a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_1 x + a_0$

where $a_0, a_1, ..., a_n$ are real numbers ($a_n \neq 0$) and n is a whole number (a nonnegative integer, $n \in \{0,1,2,3,4,5,...\}$).

$a_n x^n$ is the leading term
a_n is the leading coefficient, and
a_0 is the constant term

Which of the following are polynomials?

1. $3x^4 + x - 1$
2. $x^3 - 3x^{-1} + x^2 + 1$
3. $x^5 - 3x + \sqrt{x} - 2$
4. $2x^4 + x^3 + x^2 - 3x$
5. $\frac{2}{x^2}$
6. 5

Expressions 1, 4 and 6 are polynomials. Expression 2 is not a polynomial because there is a negative exponent -1. Expression 3 is also not a polynomial because $\sqrt{x} = x^{1/2}$. The exponent is not a whole number but a fraction. Expression 5 is not a polynomial because $\frac{2}{x^2} = 2x^{-2}$.

In the polynomial $3x^4 + x - 1$, there are expressions that are separated by plus and/or minus sign. These are called terms. Therefore, $3x^4 + x - 1$ has three terms which are $3x^4$, x and -1.

Polynomials can be classified according to the number of terms.

CLASSIFICATION	NUMBER of TERMS	EXAMPLE
Monomial	One	$5x^3y^2$ and -10
Binomial	Two	$5x^3 - y^2$ and $x - 1$
Trinomial	Three	$5x^3 - y^2 + 10$
Polynomial of n terms	n terms	$5x^3 + y^2 + x^3y^2 + 3$

Be careful not to confuse terms and factors. In a polynomial, terms are added or subtracted while factors are multiplied.

Terms consist of numerical coefficient and literal coefficient. Numerical coefficient is the numeric factor while the literal coefficient is the literal factor. Thus, in a term $3x^4$, 3 is the numerical coefficient and x^4 is the literal coefficient.

For each of the following polynomials, identify the numerical and literal coefficient.

1. $-5a$
2. x^3
3. $-y^{10}$
4. $15p^4$
5. -25
6. $10x^4y^2$

Terms	Numerical Coefficient	Literal Coefficient
$-5a$	-5	a
x^3	1	x^3
$-y^{10}$	-1	y^{10}
$15p^4$	15	p^4
-25	-25	none
$10x^4y^2$	10	x^4y^2

Now, consider the polynomial $3x^5y^4 - 2x^4y^5 - 2x^5y^4 + 5x^4y^5$. How many terms does it have?

There are four terms namely:

	Numerical Coefficient	Literal Coefficient
$3x^5y^4$	3	x^5y^4
$-2x^4y^5$	-2	x^4y^5
$-2x^5y^4$	-2	x^5y^4
$5x^4y^5$	5	x^4y^5

Observe that there are terms that have the same literal coefficients with the other terms. These are called ***like*** or ***similar terms***. Thus, $3x^5y^4$ is similar with $-2x^5y^4$. Similarly, $-2x^4y^5$ is similar with $5x^4y^5$.

Recall that when we ***add and subtract polynomials, we combine only the similar or the like terms.***

Example:

1. Simplify $9m^4 - 2m^3 + 6m^2 + 3 + 5m^4 - 7m^2 + 6$.

The terms in the polynomial can be rearranged using the commutative property in such a way that the similar terms are written beside each other.

$$9m^4 + 5m^4 - 2m^3 + 6m^2 - 7m^2 + 3 + 6$$
$$= 14m^4 - 2m^3 - m^2 + 9$$

2. Perform the operations.

a. $(7x^3 + 5x^2 - x + 6) + (x^3 + 3x^2 + 5x)$

The two polynomials can be added by combining the like terms. The terms can be regrouped.

$$(7x^3 + x^3) + (5x^2 + 3x^2) + (-x + 5x) + 6$$
$$= 8x^3 + 8x^2 + 4x + 6$$

The parentheses may also be removed right away.

$$7x^3 + 5x^2 - x + 6 + x^3 + 3x^2 + 5x$$
$$= 7x^3 + x^3 + 5x^2 + 3x^2 - x + 5x + 6$$
$$= 8x^3 + 8x^2 + 4x + 6$$

b. $(7x^3 + 5x^2 - x + 6) - (x^3 + 3x^2 + 5x)$

Recall that we can subtract a real number by adding its additive inverse $a - b = a + (-b)$. b and $-b$ are additive inverses of each other since there is 0. The same applies when subtracting polynomials.

$(7x^3 + 5x^2 - x + 6) - (x^3 + 3x^2 + 5x)$ can be rewritten as

$= (7x^3 + 5x^2 - x + 6) + [-(x^3 + 3x^2 + 5x)]$ Note that $(x^3 + 3x^2 + 5x)$ and $-(x^3 + 3x^2 + 5x)$ are additive inverses.

$= (7x^3 + 5x^2 - x + 6) + [-x^3 - 3x^2 - 5x]$ -1 factor is distributed inside $(x^3 + 3x^2 + 5x)$

$= 6x^3 + 2x^2 - 6x + 6$

TRY THESE.

A. State whether each of the following expression is a polynomial or not. Write YES or NO.

___1. $3x^2 + x + 4$

___2. $\frac{4}{a} + 9$

___3. $10x^{10} + 3x^8 + 6x^2 + 4$

___5. -15

___6. $-x^0$

___7. $5z^{-2} + z^{-1} + 4$

___4. $\frac{b^2}{2}+\frac{b}{4}+3$

___8. $x^{10} – x^9$

B. ***Indicate the number of terms in each algebraic expression, then identify their numerical and literal coefficients.***

		No. of Terms	Numerical Coefficient	Literal Coefficient
9.	$-7x^3y^3 + 6x^2y^2 + 3xy + 7$			
10.	$\frac{-1}{5}x^4y^5 - \frac{2xy}{5}$			

C. Combine and simplify.

1. 12a + 5b – 7a – 14b – 9a
2. 5x + 7y + 3z – 2x – 4y + z
3. 3a – 2b + 6c – 5a – 7b - c
4. $2x^3 – 5x^2 + 7x^3 – 4x^2 + 5x$
5. $\mathbf{5ab - \frac{3}{2}ab + \frac{3bc}{4} + \frac{1}{4}cb}$
6. –4(5y – 7) + 3(2y – 5)
7. 8(2k – 1) – (4k – 3)
8. 6(3p – 2) – (5p + 1)
9. –2(-3k + 2) – (5k – 6) – 3k - 5
10. –2(3r – 4) – 6(6 – r) + 2r - 5

NEED MORE CHALLENGE.

Combine and simplify.

1. 15a – 2(7a – 4b)
2. 2(6x + 5y) + 7(2x – y)
3. 3(a – 3) – 4(1 – a)

4. $7(p - 3q) + 5(q - p)$

5. $-2(p + 5q) + 3(5p - 4q)$

6. $3(x - 2y) - 2(3x - y) + 6(x - y)$

7. $(x + y) - 2(5x - y) - 4(3x - 2y)$

8. $2(3x + y) - 5[\ 3(x - 3y) - 4(2x - y)\ ]$

9. $7[\ (x - y) - 2(x + 3y)\ ] - 4(x - 13y)$

10. $2x - 3\ \{\ 2(5x - y) - 4[\ x - (7x - y)\ ]\ \}$

11. $3px + qy - rz - (px - qy + rz)$

12. $5p + 3q - 4r - (6q - 3p + r)$

13. $1 - \{\ 1 - [\ 1 - (1 - x)\ ]\ \}$

14. $-(5a^2 - 4a - 3) - (4a - 1) + (1 - 2a^2)$

15. $[2(x + 5y) - 3(x - y)] - 7[3x - (x + 6y)\ ]$

CONQUER THESE.

A. Perform the indicated operations.

1. $(2h^2 + 4k^2) + (-5h^2 - k^2) + (h^2 - k^2)$

2. $(2ab + 3bc) + (5ac - 5ba) + (2cb + 5ab)$

3. $(2a^2 + 3b^2 - c) + (5b^2 + 2a^2 + 5c) + (a^3 - b^2)$

4. $\frac{1}{2}xy + \left(\frac{1}{3}xy^2 - \frac{1}{4}xy\right) + \left(\frac{1}{6}xy^2 + xy\right)$

5. $(8x^3 + 7x^2) + (4x^2 + 3x - 1) + (8x^2 + 9x^3 - 4)$

6. $(3x^2 + 4x + 5) + (-8x^2 + 9x - 7)$

7. $(2b^3 - 4b^2 + 7) + (-5b^3 - 3b^2 - 11)$

8. $(5d^2 + 2d - 9) + (-3d^2 - 5d + 1)$

9. $-(3a^3 + 4a^2 - 9) + (12a^3 - 3a^2 + 15$

10. $(7x^2 + 5x^3 – 4x – 5) – (2x^3 + 5x^2 – 4x + 3)$

11. $(14 + 2x – 12x^2 + 7x^3) – (5x^3 + 4x^2 – 3x – 2)$

12. $3(2x^3 – 2x^2 + 7) – (4x^2 + x^3 – 5x)$

13. $5x^3 – 5x^2 + 14 – 2(x^3 – 6x^2 – 4)$

14. $–3(3x^2 – 5) – 12x^3 + 5x – 9$

15. $[6(2 – 3a + 5a^2) – 4(a^2 + 5)] – 2(3a^2 – 5a)$

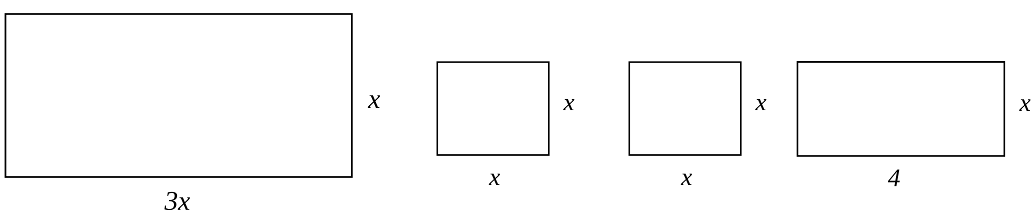
16. $(15y^2 + 8y – 10) – (2y – 4y^2 + 15) + (8y^2 – 3y)$

17. $– (–2a + 5b – 3c + 4d) – 4c + 2d – a$

18. $m – (–3y + 5w – m) – 3y – 5w$

B. Answer the following:

1. a. Express the sum of the areas of these rectangles as a polynomial.



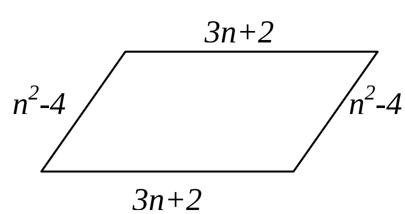
b. Find the sum of the areas when x = 3.

c. Find the sum of the areas when x = 8.

2. Use the lengths of the sides of the parallelogram to explain why the value of *n* must be greater than 2.

a. Express the perimeter as a simplified polynomial.

b. Why must the perimeter be greater than 16?



Lesson 1.2	Review of Laws of Exponents
	OBJECTIVE: To simplify expressions using the laws of real number exponents.

Recall that an exponent tells how many times a factor is used as a base. In an exponential expression 2^5 which is read as 2 to the fifth power, 2 is called the base and 5 is called the exponent. It means that we have to multiply the base 2 five times. Thus, $2^5 = 2x2x2x2x2 = 32$. The exponential value or exponential product is 32.

The definition of exponent is used in developing different rules and laws of exponents.

Product Rule

For any real number x, and for all integers m and n,

$$x^m \bullet x^n = x^{m+n}$$

Examples:

Simplify.

1. $2^5 \bullet 2^3 = 2^{5+3} = 2^8$

   $2^5 \bullet 2^3$
   $= (2 \bullet 2 \bullet 2 \bullet 2 \bullet 2)(2 \bullet 2 \bullet 2)$
   $= 2^8$

2. $b \bullet b^2 \bullet b^4 = b^{1+2+4}$
   $= b^7$

   $(b)(b \bullet b)(b \bullet b \bullet b \bullet b)$
   $= b^7$

3. $(x^3 y)(x^3 y^4 z^2)$
   $= (x^3 x^3)(y y^4)(z^2)$
   $= (x^{3+3})(y^{1+4})(z^2)$
   $= x^6 y^5 z^2$

   $(x \bullet x \bullet x \bullet y)(x \bullet x \bullet x \bullet y \bullet y \bullet y \bullet y \bullet z \bullet z)$
   $= (x \bullet x \bullet x \bullet x \bullet x \bullet x)(y \bullet y \bullet y \bullet y \bullet y)(z \bullet z)$
   $= x^6 y^5 z^2$

Quotient Rule

For any real number x except 0, and for all integers m and n,

$$x^m \div x^n = \frac{x^m}{x^n} = x^{m-n}.$$

Examples:

1. $\frac{5^4}{5^2} = 5^{4-2} = 5^2$ $\qquad$ $\frac{5 \bullet 5 \bullet 5 \bullet 5}{5 \bullet 5} = 5^2$

2. $\frac{a^5 \bullet b^6 \bullet c^2}{a \bullet b^4} = a^{5-1} \bullet b^{6-4} \bullet c^2 = a^4 b^2 c^2$

Quotient Rule results to two more interesting rule and definitions. The above examples illustrated results where exponents of the expression in the numerator is greater than the exponent of the expression in the denominator.

Using the law, $x^m \div x^n = \frac{x^m}{x^n} = x^{m-n}$, what if (a) $m = n$? or (b) what if $m < n$?

If $m = n$, then $x^m \div x^m = \frac{x^m}{x^m} = x^{m-m} = x^0 = 1$. This led to the zero exponent rule.

Zero Exponent Rule

For any real number x except 0, $x^0 = 1$.

Examples:

1. $4^0 = 1$

2. $-5(ab)^0 = -5(1) = -5$

If $m < n$, then $x^m \div x^n = \frac{x^m}{x^n} = x^{m-n}$. The difference of $m - n$ is less than 0. Thus, the difference is negative. This led to the negative exponent rule.

Negative Exponent Rule

For any real number x except 0, and for all integers m,

$$x^{-m} = \frac{1}{x^m}.$$

Examples:

1. $4^{-2} = \frac{1}{4^2}$ $\qquad$ 4^{-2} can be a result of simplifying

$$\frac{4^3}{4^5} = \frac{4 \bullet 4 \bullet 4}{4 \bullet 4 \bullet 4 \bullet 4 \bullet 4} = \frac{1}{4^2}.$$

2. $ab^{-2} = a\left(\frac{1}{b^2}\right)$

Power Rule 1. Raising a Power to a Power

For any real number x, and for all integers m and n,

$$(x^m)^n = x^{mn}.$$

Examples:

1. $(4^3)^2 = 4^{3(2)} = 4^6$

2. $(-2^2)^4 = (-2)^{2(4)} = (-2)^8 = 2^8$

3. $(-2^3)^5 = (-2)^{3(5)} = (-2)^{15} = -2^{15}$

Power Rule 2. Raising a Product to a Power

For any real number x and y, and for all integers n,

$$(xy)^n = x^n y^n.$$

Examples:

1. $(4y)^2 = 4^2 y^2 = 16y^2$

2. $(-3m)^5 = (-3)^5 m^5 = -243m^5$

3. $(2a^3b)^2 = 2^2(a^3)^2(b)^2 = 4a^6b^2$

Power Rule 3. Raising a Quotient to a Power

For any real number x and y except $y = 0$, and for all integers n,

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}.$$

Examples:

1. $\left(\frac{x^2}{5}\right)^3 = \frac{x^{2(3)}}{5^3} = \frac{x^6}{125}$

2. $\left(\frac{2a^3}{b^4}\right)^2 = \frac{2^2(a^3)^2}{(b^4)^2} = \frac{4a^6}{b^8}$

TRY THESE.

A. Write each of the following in a shorter way.

1. $20 \bullet 20$
2. $0.5 \bullet 0.5 \bullet 0.5 \bullet 0.5 \bullet 0.5$
3. $\frac{1}{4} \bullet \frac{1}{4} \bullet \frac{1}{4}$
4. $2b \bullet 2b \bullet 2b \bullet 2b \bullet 2b$
5. $(4-8) \bullet (4-8) \bullet (4-8) \bullet (4-8)$
6. $(p+q+r) \bullet (p+q+r) \bullet (p+q+r)$
7. $(-2) \bullet (-2) \bullet (-2) \bullet (-2) \bullet (-2) \bullet (-2)$
8. $4 \bullet 4 \bullet 4 \bullet 5 \bullet 5 \bullet 5$
9. $2 \bullet 2 + 3 \bullet 3 \bullet 3 - 4 \bullet 4 \bullet 4 \bullet 4$
10. $x \bullet x \bullet y \bullet z \bullet z \bullet z \bullet z$

B. Expand.

1. 8^5
2. $(-4)^4$
3. $(2p)^2$
4. $(m-1)^3$
5. $3^4 + m^4$

C. Find the exact value of the following expressions:

1. 5^{-2}
2. 3^{-3}
3. $\frac{8^{-2}}{8^{-3}}$
4. $(4^{-2}\ 2^2)^{-1}$
5. $((7^{-3})^{-1})^{-2}$
6. $((7^{-3})^{-1})^2$
7. $((1^{-1})^{-3})^{-5}$
8. $\left(\frac{1^{-4}.1^4}{1^3.1^3}\right)^{-10}$
9. $\left(\frac{4^{-2}.9^{-3}}{2^{-4}.3^{-6}}\right)^{-5}$
10. $\left(\frac{(2^{-4}.3^{-2})(4^2.9^{-4})}{3^{-10}.2^0}\right)$

NEED MORE CHALLENGE.

Simplify and express the results without zero or negative exponents.

1. $\dfrac{5^{-1}x^{-2}y^{3}z^{0}}{5^{-2}x^{-3}y^{-3}z^{-1}}$

2. $\dfrac{m^{-4-2}}{m^{2}}$

3. $\left(\dfrac{x^{0}}{y^{2}}\right)^{-1}$

4. $\dfrac{a^{-1}+b^{-1}}{a^{-1}-b^{-1}}$

5. $\dfrac{a^{-2}-b^{-2}}{a^{-2}b^{-2}}$

6. $\dfrac{x^{-2}y^{-1}+x^{-1}y^{-2}}{x^{-1}y^{-1}}$

7. $\left(\dfrac{c^{-5}e^{-4}l^{2}}{l^{-1}a^{-1}c^{-6}e^{-5}l^{-1}}\right)^{-1}$

8. $\left(\left(\dfrac{v^{-1}w^{0}}{w^{2}v^{-2}}\right)^{-1}\right)^{-2}$

9. $\dfrac{y^{-6}-x^{-6}}{(x^{2}-y^{2})x^{-6}y^{-6}}$

10. $\dfrac{x^{-1}y^{-2}+3x^{-2}y^{-1}}{y^{-2}-9x^{-2}}$

11. $-3\ (x + 2)\ (x - 2)^{-2} + (x - 2)^{-1}$

12. $2\ (2x - 3)^{-2}\ (3x + 4)^{-2} + 3\ (2x - 3)^{-3}\ (3x + 4)^{-1}$

13. $2(x + y)(x^{2} - y^{2})^{-1} + (x^{2}+xy+ y^{2})(x^{3} - y^{3})^{-1}$

14. $\dfrac{a^{-2}-a^{-3}y^{-2}}{a^{-3}y^{-2}-y^{-2}}$

15. $(x^{0} - 2)\ (x^{0} + 2)$

CONQUER THESE.

A. Simplify.

1. $x^{2a}x^4$

2. $x^{3a}x^{2b}$

3. x^5x^{2a-4}

4. $(a^{n+1}b^{m+2})^3$

5. $(x^a y^{a-3})^3$

6. $(c^3d)^a(cd^7)^a$

7. $x^2(x^{a+2}y^3)$

B. Answer the following:

1. Find the volume of a cube with sides of length 3x.

2. Suppose the cube has sides of length 5x. What would its volume be?

3. Suppose the cube has sides of length 9x. What would its volume be?

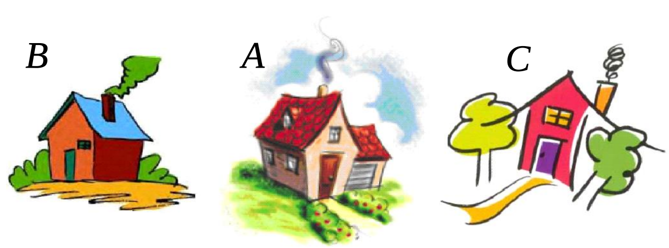
4. How do the volumes of each of the cubes above change as the dimension doubles? Triples?

<table>
<tr><td rowspan="2">Lesson 1.2</td><td>Segments and Rays</td></tr>
<tr><td>OBJECTIVES:
1. To use the concept of coordinate plane to locate positions
2. To use the concept of coordinate plane to find distance</td></tr>
</table>

Betweenness

The concepts of rays and segments are particularly important for the study of distance. We defined segment in the previous lesson as *the union of two points on the line and all the points between them*. The notion of *between* should be clear. By between we refer to only those points on the same line. Consider the diagram below.

Which among the three houses are between the other two? House A may claim it is between Houses B and C, but likewise House B may make the same claim. It is confusing. But if we consider the diagram below, it is clear which house is between the other two.



The concept of *between* applies only to those points that are on the same line. The term between in geometry is defined this way-

*A point B is **between** points A and C, written as B-A-C, if*

- *a.* *A, B and C are collinear;*
- *b.* $AB + BC = AC$

The definition is two-folds and it includes an equation. Note that AB, BC and AC are new notations. The notation $\overline{AB}$ refers to the line segment with endpoints A and B. On the other hand, "AB" denotes distance from points A to B.

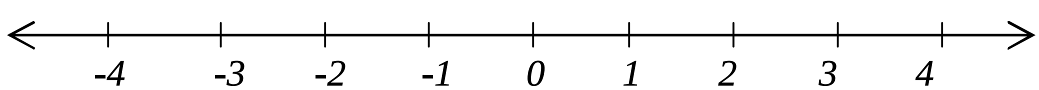
Distance

Distance in the real world is the measure that tells how far apart two objects are. There is no difference in geometry. In fact we may consider this concept of distance in geometry the very basis of the concept of distance we know. The first thing we note is that whenever you have two distinct locations, there is distance. Remember that a point connotes location or position. Thus, each pair of points has distance (as each pair of locations or positions has distance).

<u>Distance Postulate</u>

To every pair of different points there corresponds a unique positive number.

The unique positive real number mentioned in the distance postulate is distance. Now placing these points on the line allows us to compute for distance. Examine the diagram below.



The diagram shows the familiar picture of a line, but this time with numbers written below. This is a *number line*. This concept was introduced when we were studying about system of numbers. It shows the relationship of numbers; numbers at the right of zero are like mirror images of the numbers on the left of zero but with negative signs. As a number gets father from zero to the right, the number gets bigger and bigger in a constant fashion. This number line is no different from the number line we study in elementary algebra. But what really is the idea behind the number line?

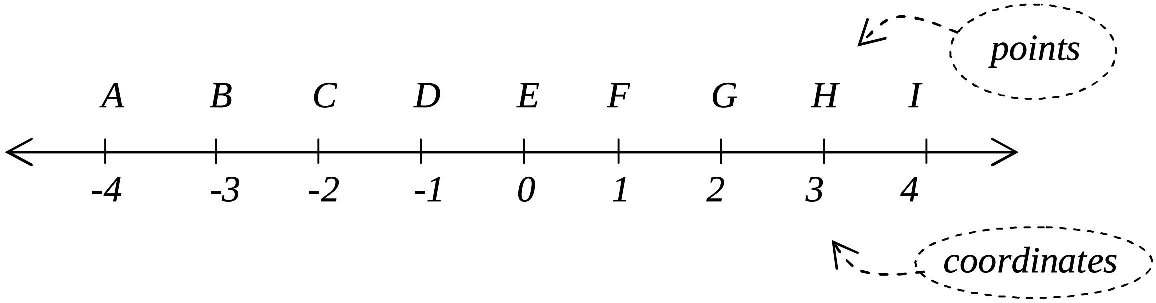
Remember that a line consists of infinitely many points that are so condensed. This characteristic of line is the same as that of numbers. There are as well infinitely many numbers and these numbers are so "condensed" that between two numbers, however close they are, there is still a number in between. Given this, we can pair the points on the line and the real numbers, such that each point on the line has a unique "partner" in the real number system. This is called o*ne to one correspondence*. This fact is formally known in geometry as the *Ruler Postulate*.

Ruler Postulate

The points on the line can be placed in correspondence with real numbers in such a way that

a. *for every point on the line, there corresponds exactly one real number;*
b. *for every real number, there corresponds exactly one point on the line; and*
c. *the distance between two points is the absolute value of the difference of the corresponding numbers.*

We can say that this postulate has two parts: the correspondence part and the formula part. The former simply states the unique pairing of points on the line and real numbers. The diagram below shows the number line with points labeled accordingly. The corresponding real number is called "coordinate".



In the diagram above, the point D has coordinate -1. The number 3 is the coordinate of the point H. Can you plot the point P with coordinate pi?

The latter part of ruler postulate gives us the formula in finding the distance between two points on the line. The distance formula is given by the absolute value of the difference of the coordinates of the given points. Hence, in the diagram above, the distance between the points B and G, denoted by BG, is given by

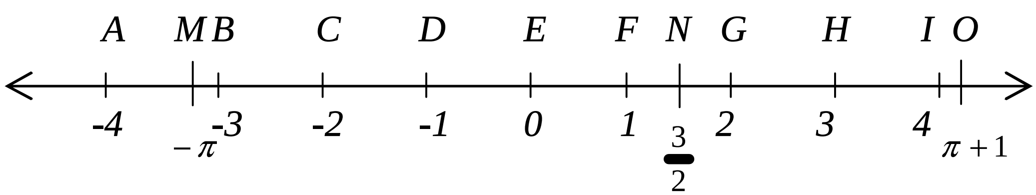
$$\begin{aligned} BG &= |-3-2| \\ &= |-5| \\ &= 5 \end{aligned}$$

The formula gives us the right distance because in the diagram, the point G is really 5 units away from B. But what if we have interchanged the coordinates? Would we get the same answer?

Recall that absolute value is the “positive number value” of a real number. When we interchange the coordinates the differences we get differ only in signs. Getting absolute value gives us then the same answer. Thus, it makes no harm to interchange the coordinates of the points.

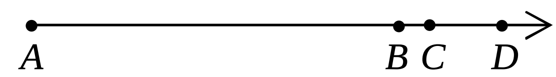
Exercise: Finding Distance

Using the number line below, determine the following distances.



1.	AC	6.	NG
2.	MD	7.	CE
3.	EO	8.	NM
4.	OM	9.	NO
5.	NC	10.	AO

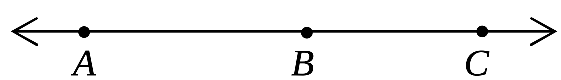
A ray was defined as part of a line that extends indefinitely in only one direction. Consider the diagram of $\overrightarrow{AB}$ below, which shows some more points C and D.



Note that for any point at the right of B, B is between that point and the point A. For example, A-B-C and A-B-D. A ray can also be defined this way. Can you define $\overrightarrow{AB}$ using the concept of between?

> *<u>Ray AB</u>, denoted by $\overrightarrow{AB}$, is the union of $\overline{AB}$ and all the points C for which A-B-C. The point A is called the endpoint of the ray.*

Would $\overrightarrow{BA}$ have the same definition as $\overrightarrow{AB}$? Note that this time, the endpoint is B. Thus, these two rays are different. Another concept related to ray is *opposite rays*. As the term suggest, opposite rays are rays that point to opposite directions. In the diagram below, BA and BC are opposite rays.



The endpoint B is common to the two rays. It divides the line into two parts. Apparently, the point B is not in the exact middle of $\overleftrightarrow{AC}$ in the diagram. Had it been in the middle, did it divide the line into two equal parts? Can a line be divided?

As the line consists of infinitely many points, we cannot say it has a middle point. The line has no length; we cannot measure it. On the other hand, a line segment has fixed length. Thus, a segment has a middle point we call *midpoint*.

> *A point B is called the <u>midpoint</u> of $\overline{AC}$ if*
> - *a.* A-B-C and
> - *b.* $AB = BC$

How many midpoints does a segment have? The theorem below answers this question.

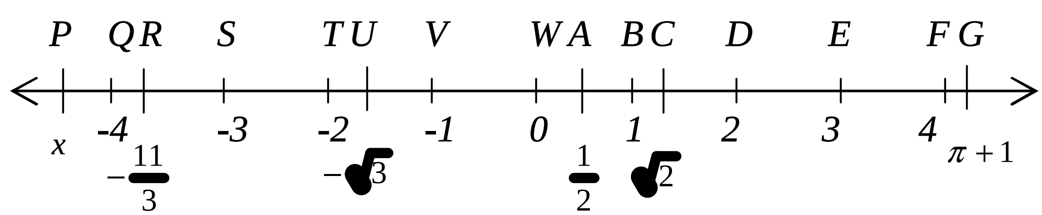
__The Midpoint Theorem__

Every line segment has exactly one midpoint.

The proof of this theorem will be discussed at a later time. It is interesting to know though how it will be proved. We might say it is so obvious. Obvious though is not an answer people will accept.

More Exercises

I. Using the diagram below, identify what it is asked in each item.



1. The coordinate o U
2. The coordinate of R
3. The point with coordinate of x
4. The point with coordinate of 0.5
5. The point with coordinate $\sqrt{16}$

II. Using the same diagram above, find the following distances.

1.	QW	6.	CU
2.	RA	7.	BR
3.	RF	8.	AG
4.	PW	9.	GU
5.	PU	10.	CT

III. Solve the following problems completely.

1. M is the midpoint of $\overline{LN}$. The coordinate of L and N are -14 and 4 respectively. Find the coordinate of K and the distance LM.

2. Suppose the endpoints of the given line segment correspond to –12 and 2 respectively, what is the coordinate of its midpoint?

3. A, B and C are three points of a line with $AC = AB = 10$. The coordinate of C is 8 and the coordinate of A is greater than the coordinate of B. Find the coordinates of A and B.

4. Suppose the endpoints of the given line segment correspond to –12 and 4 respectively, what is the distance AB and AC given that $AB = 3x$ and $BC = x$?

5. If the coordinates of the endpoints are –18 and –6 respectively, what is the coordinate of B if AB is twice BC?

IV. Construct a number line following the indications in each item.

1. A has coordinate $-\frac{3}{4}$.
2. B has coordinate $\pi - 5$.
3. C is the origin.
4. D has greater coordinate
   and 3 units away from B
5. E is exactly 4 units to the left of A.

Second Year AS Geometry WORKLOAD

GENERAL DIRECTIONS: Read and study the lesson about 'Segments and Rays' and answer all the EXERCISES. It is required that you submit this on July 2, 2009.