**THIRD YEAR ALGEBRA WORKLOAD**

**GENERAL DIRECTIONS**: Read and study the lessons 1.1 and 1.2. Answer only ' Try These' and 'Need more Challenge 1-15' from lesson 1.1. Show complete solution in size 1 paper and box final answer/s. It is required that you submit these on July 2, 2009.

| Lesson 1.1 | General Term and Elements of a Sequence |
|---|---|
| | **OBJECTIVES:**<br>1. To define sequence function.<br>2. To give the elements of a sequence.<br>3. To give the general tem of a sequence. |

Are you familiar with this?

13 – 3 – 2 – 21 – 1 – 1 – 8 – 5
O, Draconian devil!
Oh, lame saint!

This is from Dan Brown's novel *The Da Vinci Code* which is scrawled in invisible ink on the floor of the Louvre in Paris by a dying man with a passion for secret codes. The numbers may puzzle Robert Langdon for a while but any mathematician will recognize them at one. They are the first eight numbers of the Fibonnaci sequence written in jumbled order.

Sequences of numbers are also often encountered in IQ exams.

For instance,

What is the next number in the sequence 2, 4, 6, 8, 10?

Can you make your own sequence? How were you able to make your own?

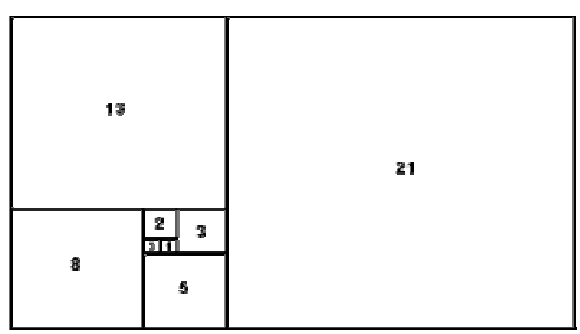
***As long as there is a defined rule of pattern, you can make your own sequence.***

A sequence having a last number is called a **finite** sequence otherwise it is called an **infinite** sequence.

The **Fibonnaci numbers** are a sequence of numbers named after Leonardo of Pisa known as Fibonnaci.

$$F_n = \begin{cases} 0 & n = 0 \\ 1 & \text{if } n = 1 \\ F_{n-1} + F_{n-2} & n > 1 \end{cases}$$

After rearranging the numbers above, what now is the correct Fibonnaci sequence?



**Sequence function** is a function whose domain is a subset of positive integers. Thus, the **domain** of a finite sequence function is **{1, 2, 3,4,5,..., n}** while the domain of an infinite sequence function is **all positive integers**. The element one in the domain pertains to first, 2 as second, and so on.

Here are some examples:

1. $f(n) = 2n$ Domain: {1,2,3,4,5}

   f(1) = 2, f(2) = 4, f(3) = 6, f(4) = 8, and f(5) = 10. Thus the first five elements of the sequence function are 2, 4, 6, 8, and 10. The range of the given sequence function is {2,4,6,8,10}.

2. $g(n) = \frac{1}{n}$ Domain: { 1, 2, 3, ... }

   $g(1) = \frac{1}{1} = 1$, $g(2) = \frac{1}{2}$, $g(3) = \frac{1}{3}$ ... Thus, the range is $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$. The elements of the sequence function or simply the elements of the sequence are $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$.

3. Give the elements of the infinite sequence $h(n) = \frac{2n-1}{n^2}$.

The first element of the sequence is usually denoted by $a_1$, second element as $a_2$, $a_n$ as the nth element. The nth element is also called as the general element. Like in our second example above, $a_1$ is 1, $a_2$ is $\frac{1}{2}$ and the general element is $\frac{1}{n}$.

**EXERCISES:**

I. Write the first five elements for each of the following general elements.

1. $a_n = 2n + 3$

2. $a_n = \frac{n+2}{n(n+1)}$

3. $a_n = (-1)^n \frac{1}{3^{n-1}}$

4. $a_n = (-1)^{n-1} x^{2n+1}$

II. Write the first 10 elements of the sequence whose general element is given.

1.
$$a_n = \begin{cases} \frac{2}{n+1} & \text{if n is odd} \\ 2 & \text{if n is even} \end{cases}$$

2.
$$a_n = \begin{cases} 1 & \text{if n is odd} \\ \frac{4}{(n+2)^2} & \text{if n is even} \end{cases}$$

3.
$$a_n = \begin{cases} n & \text{If n is odd} \\ \frac{1}{2}n & \text{If n is even and is not divisible by 4} \\ \frac{1}{2}(a_{n-2} + a_{n-1}) & \text{If n is even and is divisible by 4} \end{cases}$$

III. For each of the following sequence, give the general element.

1. 1, 3, 5,7, 9, 11, ..., n

2. $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots, n$

3. $\frac{1}{2}, -\frac{x^2}{4}, \frac{x^4}{6}, -\frac{x^6}{8}, \frac{x^8}{10}, \ldots, n$

**TRY THESE**

**A. For each of the following sequence, determine the first five elements.**

1. $a_n = \frac{2n-1}{n^2}$

2. $a_n = \frac{n+2}{n(n+1)}$

3. $a_n = (-1)^n \frac{1}{2^{n-1}}$

**B. Write the first 12 elements of the sequence for which**

$$a = \begin{cases} n & \rightarrow \text{if n is odd} \\ n & \rightarrow \text{if n is even and not exactly divisible by 4} \\ \frac{1}{2}(n + a_{n-2}) & \rightarrow \text{if n is even and exactly divisible by 4} \end{cases}$$

**C. Determine the general term $a_n$ for each of the following sequence.**

1. 1, 8, 27, 64, 125, ...

2. $-x^2$, $x^4$, $-x^6$, $x^8$, $-x^{10}$, $x^{12}$,...

3. $x^2$, $\frac{1}{x^4}$, $x^6$, $\frac{1}{x^8}$, $x^{10}$, $\frac{1}{x^{12}}$ ,...

**NEED MORE CHALLENGE?**

Derive the general term given the elements of the sequence.

1. 1, 2, 3, 4, 5, 6, 7, . . .

2. 2, 4, 6, 8, 10, 12, . . .

3. 1, 3, 5, 7, 9, 11, . . .

4. 1, 4, 9, 16, 25, 36, 49, . . .

5. 4, 9, 16, 25, 36, 49, 64, . . .

6. 2, 5, 10, 17, 26, 37, 50, . . .

16. –5, –9, –13, –17, –21, –25, . . .

17. –1, 1, –1, 1, –1, 1, –1, . . .

18. $-\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, -\frac{1}{32}, \frac{1}{64}, . . .$

19. 2, 5, 8, 11, 14, 17, . . .

20. 4, 10, 18, 28, 40, 54, . . .

21. –1, 4, –27, 256, –3125, . . .

7. 1, 4, 7, 10, 13, 16, 19, . . .

8. –3, 0, 3, 6, 9, 12, 15, . . .

9. 0, 1, 4, 9, 16, 25, 36, . . .

10. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \ldots$

11. $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \frac{1}{243}, \ldots$

12. $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \ldots$

13. $1, \frac{1}{8}, \frac{1}{27}, \frac{1}{64}, \frac{1}{125}, \ldots$

14. $\frac{4}{3}, \frac{5}{4}, \frac{6}{5}, \frac{7}{6}, \frac{8}{7}, \frac{9}{8}, \ldots$

15. 4, 14, 24, 34, 44, 54, . . .

22. 1, 1, 1, 1, 1, 1, 1, . . .

23. –6, –4, 0, 6, 14, 24, 36, . . .

24. $1, \frac{5}{2}, 5, \frac{17}{2}, 13, \frac{37}{2}, 25, \ldots$

25. $2, \frac{5}{2}, \frac{10}{3}, \frac{17}{4}, \frac{26}{5}, \frac{37}{6}, \frac{50}{7}, \ldots$

26. $x, x^2, x^3, x^4, x^5, \ldots$

27. 2x, 4x, 8x, 16x, 32x, . . .

28. $\frac{x}{2}, \frac{x}{3}, \frac{x}{4}, \frac{x}{5}, \ldots$

29. $\frac{x-3}{x+2}, \frac{x}{x+4}, \frac{x+3}{x+6}, \frac{x+6}{x+8}, \frac{x+9}{x+10} \ldots$

30. $\frac{x}{2}, \frac{2x^2}{4}, \frac{3x^3}{8}, \frac{4x^4}{16}, \frac{5x^5}{32}, \ldots$

| Lesson 1.2 | Summation Notation and Series |
|---|---|
| | **OBJECTIVES:**<br>1. To write the sum using sigma notation.<br>2. To find the sum of the series. |

We use summation notation $\sum$ to facilitate writing the sum of the elements of a sequence. The Greek symbol $\sum$ (capital sigma) corresponds to letter S. For instance, $\sum_{n=1}^{10} 2n$ is the shorthand way of writing the sum of the first ten even numbers.

$$\sum_{n=1}^{10} 2n$$

**2n** is called the summand (or the general term of the series)
**n** is the index of summation
**1 and 10** are the limits of summation, 1 being the lower limit and 10, the upper limit

The index of summation is considered as a dummy variable since $\sum_{n=1}^{10} 2n$ has the same representation of $\sum_{K=1}^{10} 2k$.

$\sum_{n=1}^{10} 2n$ and $\sum_{K=1}^{10} 2k$ are both equal to the expanded form

$$2(1) + 2(2) + 2(3) + 2(4) + \ldots + 2(10)$$

Generally, $\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \ldots + a_n$ and $\sum_{i=1}^{\infty} a_i = a_1 + a_2 + a_3 + \ldots$

Other examples:

**Expand.**

1. $$\sum_{m=1}^{5} \frac{1}{m} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$$

2. $$\sum_{j=3}^{6} j^3 = 3^3 + 4^3 + 5^3 + 6^3$$

3. $$\sum_{n=1}^{4} (-1)^n x^{2n} = (-1)^1 x^{2(1)} + (-1)^2 x^{2(2)} + (-1)^3 x^{2(3)} + (-1)^4 x^{2(4)}$$
$$= -x^2 + x^4 - x^6 + x^8$$

In summation sentence, the addends serve as the elements of a sequence where the summand is the general term of the sequence.

When we look for the sum of the elements in a sequence, we call it a **series**. Finite series is the sum of the elements of a finite sequence. Infinite series is the sum of the elements of an infinite series.

Write the following series with sigma notation.

1. $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}=\sum_{p=1}^{6}\frac{1}{p}$ since there are 6 elements and the leftmost element is always regarded to be the first element. Any letter can be used as an index.

2. $-1+5-9+13-17=\sum_{i=1}^{5}(-1)^{i}(4i-3)$

3. $\frac{1}{3}x^3-\frac{1}{9}x^5+\frac{1}{27}x^7-\frac{1}{81}x^9=\sum_{k=1}^{4}(-1)^{k-1}x^{2k+1}$

**EXERCISES:**

A. Find the sum of the series.

1. $\sum_{i=1}^{5}(4i-3)$

2. $\sum_{j=2}^{6}\frac{j}{j-1}$

3. $\sum_{K=1}^{4}\frac{(-1)^{k+1}}{k}$

4. $\sum_{m=0}^{4}(-1)^{m-1}x^{2m-1}$

B. Write the series with sigma notation.

1. $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\frac{1}{243}$

2. $1-\frac{25}{4}+\frac{125}{16}-\frac{625}{64}$

3. $\frac{1}{2}-\frac{x^2}{4}+\frac{x^4}{6}-\frac{x^6}{8}+\frac{x^8}{10}$

4. $-x+\frac{1}{2}x^3-\frac{1}{3}x^5+\frac{1}{4}x^7-\frac{1}{5}x^9$

**TRY THESE**

I Write in expanded form and evaluate.

1. $\sum_{i=1}^{6} 2i$

2. $\sum_{i=3}^{8} (i+1)$

3. $\sum_{i=1}^{5} (3i-1)$

4. $\sum_{i=1}^{3} (i^2+2)$

5. $\sum_{i=1}^{4} \frac{i}{i+1}$

6. $\sum_{k=1}^{4} k(k+3)$

7. $\sum_{k=2}^{5} \frac{1}{2k}$

8. $\sum_{k=3}^{7} (k-3)(k+2)$

9. $\sum_{k=1}^{6} (1)^k$

10. $\sum_{k=1}^{6} (-1)^k$

11. $\sum_{i=0}^{3} \frac{(-1)^i}{i+1}$

12. $\sum_{k=1}^{5} (-1)^k$

13. $\sum_{i=1}^{5} (-1)^i\ i$

14. $\sum_{k=2}^{7} \frac{2k+1}{k-1}$

15. $\sum_{i=1}^{4} 2^i$

II Use summation notation to express the following series.(Answers may vary).

1. 1 + 3 + 5 + 7 + 9 + 11

2. 0 + 3 + 6 + 9 + 12 + 15

3. 4 + 9 + 16 + 25 + 36 + 49

4. 2 + 5 + 10 + 17 + 26

5. $1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \frac{1}{125}$

6. $\frac{2}{3} + \frac{3}{8} + \frac{4}{15} + \frac{5}{24}$

7. $-1 + 1 - 1 + 1 - 1 + 1$

8. $-\frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32}$

9. $x + x^2 + x^3 + x^4 + x^5 + x^6$

10. 2x + 4x + 8x + 16x + 32x + 64x

11. $\frac{x}{2} + \frac{x}{3} + \frac{x}{4} + \frac{x}{5} + \frac{x}{6}$

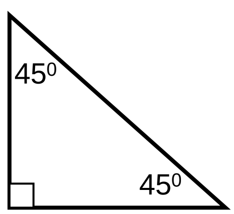
12. $\frac{x+1}{x-1} + \frac{x+2}{x-2} + \frac{x+3}{x-3} + \frac{x+4}{x-4}$

**Third Year Trigonometry**
**WORKLOAD**

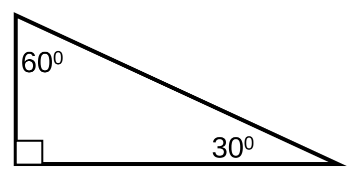
**GENERAL DIRECTIONS**: Read and study the lesson below and answer all the EXERCISES. It is required that you submit these on July 2, 2009.

| **Lesson 1.2** | **Special Right Triangles** |
|---|---|
| | **OBJECTIVES:**<br>1. Identify and describe the two special right triangles<br>2. Apply the Pythagorean Theorem on special right triangles<br>3. Solve some problems involving special right triangles |

There are two special right triangles that are very useful in Trigonometry. These are the $45^0$ x $45^0$ right triangle and the $30^0$ x $60^0$ right triangle. The uses and purposes of these right triangles have been around since the Middle Ages. Primarily, they are used for architectural designs. When the Pythagorean Theorem is applied on them, one can't help but notice some patterns in the measurements of their sides.



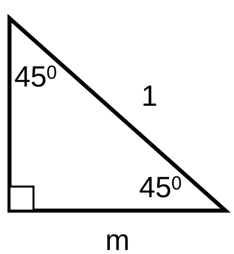

$45^0$ x $45^0$ Right Triangle

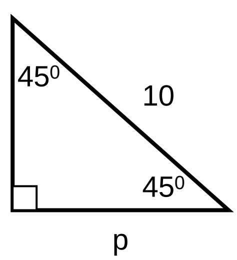


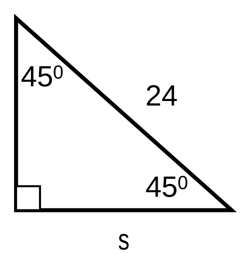

$30^0$ x $60^0$ Right Triangle

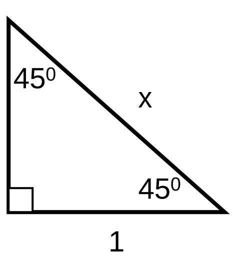
**Try these.**

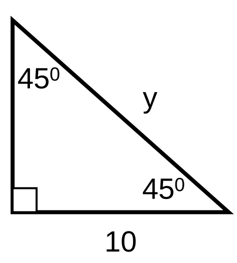
Solve for the measurements of the missing sides using the Pythagorean Theorem.

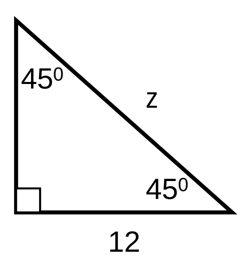



What are the measurements of the missing sides?

m = ______ p = _______ s = _______

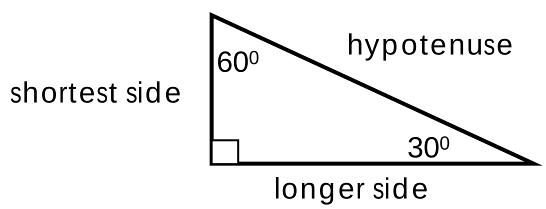
x = ______ y = _______ z = _______

**Error! Bookmark not defined.**

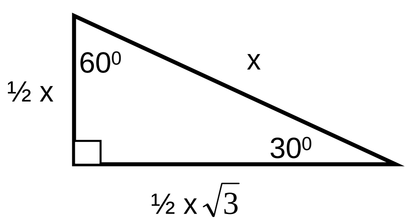
How are the sides of the $45^0$ x $45^0$ right triangles related to the hypotenuse?
You might have observed the following:

1. **The measurement of the side of the $45^0$ x $45^0$ right triangle is equal to half the measurement of its hypotenuse times the $\sqrt{2}$ .**
2. **The measurement of the hypotenuse of the $45^0$x $45^0$ right triangle is equal to the side times $\sqrt{2}$ .**

**Now, let us study the $30^0$ x $60^0$ right triangle.**




**Remember:**
In any triangle, the shortest side is always opposite the smallest angle. Therefore, in a $30^0$ x $60^0$ right triangle the shortest side is opposite the $30^0$ angle.

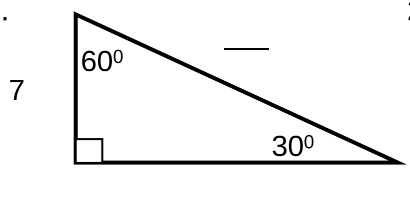



**In a $30^0$ x $60^0$ right triangle:**

1. **The measurement of the shortest side is always half the measurement of the hypotenuse.**
2. **The measurement of the longer side is equal to the measurement of the shortest side times $\sqrt{3}$ .**

**Can you do these?**

Find the lengths of the missing sides of the right triangles?

1.
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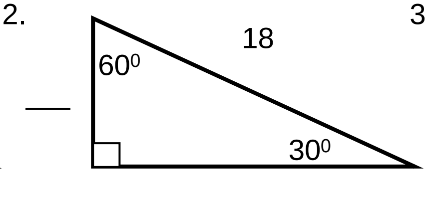


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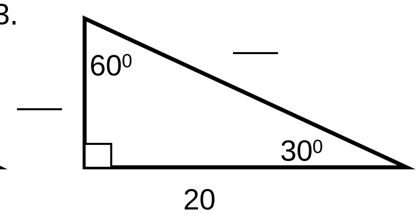


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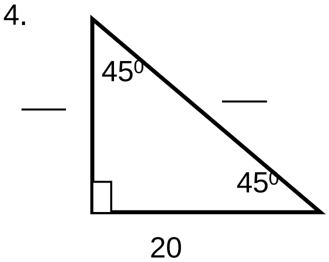


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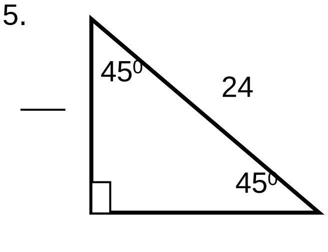
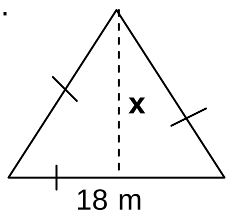


**Need more challenge?**

A. Find the length of the following segments.

1.

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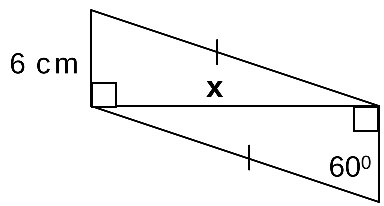


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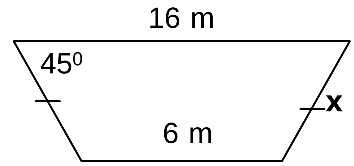


4. What is the area of a square with a diagonal of 12 cm?

______________________________

5. A ladder 15 ft. long rests on a wall. How far is the top end of the ladder from the ground if the other end makes an angle of $30^0$ against the ground?

______________________________

**B. Complete the following table.**

| Type of triangle | Length of Hypotenuse | Length of the Side | Legth of the Side |
|---|---|---|---|
| $45^0 \times 45^0 \times 90^0$ | **18** | ________________ | ________________ |
| $45^0 \times 45^0 \times 90^0$ | ________________ | **20** | ________________ |
| $30^0 \times 60^0 \times 90^0$ | **40** | ________________ **(shortest side)** | ________________ |
| $30^0 \times 60^0 \times 90^0$ | ________________ | **9** **(shortest side)** | ________________ |
| $30^0 \times 60^0 \times 90^0$ | ________________ | ________________ | **24** |