

## Third Year Algebra WORKLOAD

**GENERAL DIRECTIONS:** Read and study the lessons 1.1 – 1.3 and answer WORKSHEETS 1, 2 & 3. It is required that you submit these on July 2, 2009.

<b>Lesson</b> <b>1.1</b>	<b>Review of Laws of Exponents</b>
	<b>OBJECTIVE:</b> To simplify expressions using the laws of real number exponents.

Recall that an exponent tells how many times a factor is used as a base. In an exponential expression  $2^5$  which is read as 2 to the fifth power, 2 is called the base and 5 is called the exponent. It means that we have to multiply the base 2 five times. Thus,  $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$ . The exponential value or exponential product is 32.

The definition of exponent is used in developing different rules and laws of exponents.

### Product Rule

For any real number  $x$ , and for all integers  $m$  and  $n$ ,

$$x^m \bullet x^n = x^{m+n}$$

Examples:

Simplify.

1.  $2^5 \bullet 2^3 = 2^{5+3} = 2^8$

$$\begin{aligned} &2^5 \bullet 2^3 \\ &= (2 \bullet 2 \bullet 2 \bullet 2 \bullet 2)(2 \bullet 2 \bullet 2) \\ &= 2^8 \end{aligned}$$

2.  $b \bullet b^2 \bullet b^4 = b^{1+2+4}$   
 $= b^7$

$$\begin{aligned} &(b)(b \bullet b)(b \bullet b \bullet b \bullet b) \\ &= b^7 \end{aligned}$$

3.  $(x^3y)(x^3y^4z^2)$   
 $= (x^3x^3)(yy^4)(z^2)$   
 $= (x^{3+3})(y^{1+4})(z^2)$   
 $= x^6y^5z^2$

$$\begin{aligned} &(x \bullet x \bullet x \bullet y)(x \bullet x \bullet x \bullet y \bullet y \bullet y \bullet y \bullet z \bullet z) \\ &= (x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x)(y \bullet y \bullet y \bullet y \bullet y \bullet y)(z \bullet z) \\ &= x^6y^5z^2 \end{aligned}$$

### Quotient Rule

For any real number  $x$  except 0, and for all integers  $m$  and  $n$ ,

$$x^m \div x^n = \frac{x^m}{x^n} = x^{m-n}.$$

Examples:

$$1. \quad \frac{5^4}{5^2} = 5^{4-2} = 5^2 \qquad \frac{5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5} = 5^2$$

$$2. \quad \frac{a^5 \cdot b^6 \cdot c^2}{a \cdot b^4} = a^{5-1} \cdot b^{6-4} \cdot c^2 = a^4 b^2 c^2$$

Quotient Rule results to two more interesting rule and definitions. The above examples illustrated results where exponents of the expression in the numerator is greater than the exponent of the expression in the denominator.

Using the law,  $x^m \div x^n = \frac{x^m}{x^n} = x^{m-n}$ , what if (a)  $m = n$ ? or (b) what if  $m < n$ ?

If  $m = n$ , then  $x^m \div x^m = \frac{x^m}{x^m} = x^{m-m} = x^0 = 1$ . This led to the zero exponent rule.

### Zero Exponent Rule

For any real number  $x$  except 0,  $x^0 = 1$ .

Examples:

$$1. \quad 4^0 = 1$$

$$2. \quad -5(ab)^0 = -5(1) = -5$$

If  $m < n$ , then  $x^m \div x^n = \frac{x^m}{x^n} = x^{m-n}$ . The difference of  $m - n$  is less than 0. Thus, the difference is negative. This led to the negative exponent rule.

### Negative Exponent Rule

For any real number  $x$  except 0, and for all integers  $m$ ,

$$x^{-m} = \frac{1}{x^m}.$$

Examples:

1.  $4^{-2} = \frac{1}{4^2}$        $4^{-2}$  can be a result of simplifying

$$\frac{4^3}{4^5} = \frac{4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4} = \frac{1}{4^2}.$$

2.  $ab^{-2} = a\left(\frac{1}{b^2}\right)$

**Power Rule 1. Raising a Power to a Power**

For any real number  $x$ , and for all integers  $m$  and  $n$ ,

$$(x^m)^n = x^{mn}.$$

Examples:

1.  $(4^3)^2 = 4^{3(2)} = 4^6$

2.  $(-2^2)^4 = (-2)^{2(4)} = (-2)^8 = 2^8$

3.  $(-2^3)^5 = (-2)^{3(5)} = (-2)^{15} = -2^{15}$

**Power Rule 2. Raising a Product to a Power**

For any real number  $x$  and  $y$ , and for all integers  $n$ ,

$$(xy)^n = x^n y^n.$$

Examples:

1.  $(4y)^2 = 4^2 y^2 = 16y^2$

2.  $(-3m)^5 = (-3)^5 m^5 = -243m^5$

3.  $(2a^3b)^2 = 2^2 (a^3)^2 (b)^2 = 4a^6b^2$

**Power Rule 3. Raising a Quotient to a Power**

For any real number  $x$  and  $y$  except  $y = 0$ , and for all integers  $n$ ,

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}.$$

Examples:

$$1. \left(\frac{x^2}{5}\right)^3 = \frac{x^{2(3)}}{5^3} = \frac{x^6}{125}$$

$$2. \left(\frac{2a^3}{b^4}\right)^2 = \frac{2^2(a^3)^2}{(b^4)^2} = \frac{4a^6}{b^8}$$

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**WORKSHEET 1**

A. Write each of the following in a shorter way.

1.  $20 \bullet 20$

2.  $0.5 \bullet 0.5 \bullet 0.5 \bullet 0.5 \bullet 0.5$

3.  $\frac{1}{4} \bullet \frac{1}{4} \bullet \frac{1}{4}$

4.  $2b \bullet 2b \bullet 2b \bullet 2b \bullet 2b$

5.  $(4-8) \bullet (4-8) \bullet (4-8) \bullet (4-8)$

6.  $(p+q+r) \bullet (p+q+r) \bullet (p+q+r)$

7.  $(-2) \bullet (-2) \bullet (-2) \bullet (-2) \bullet (-2) \bullet (-2)$

8.  $4 \bullet 4 \bullet 4 \bullet 5 \bullet 5 \bullet 5$

9.  $2 \bullet 2 + 3 \bullet 3 \bullet 3 - 4 \bullet 4 \bullet 4 \bullet 4$

10.  $x \bullet x \bullet y \bullet z \bullet z \bullet z \bullet z$

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B. Expand.

1.  $8^5$

2.  $(-4)^4$

3.  $(2p)^2$

4.  $(m-1)^3$

5.  $3^4 + m^4$

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C. Find the exact value of the following expressions:

1.  $5^{-2} = \underline{\hspace{2cm}}$

6.  $((7^{-3})^{-1})^2 = \underline{\hspace{2cm}}$

2.  $3^{-3} = \underline{\hspace{2cm}}$

7.  $((1^{-1})^{-3})^{-5} = \underline{\hspace{2cm}}$

3.  $\frac{8^{-2}}{8^{-3}} = \underline{\hspace{2cm}}$

8.  $\left(\frac{1^{-4} \cdot 1^4}{1^3 \cdot 1^3}\right)^{-10} = \underline{\hspace{2cm}}$

4.  $(4^{-2} 2^2)^{-1} = \underline{\hspace{2cm}}$

9.  $\left(\frac{4^{-2} \cdot 9^{-3}}{2^{-4} \cdot 3^{-6}}\right)^{-5} = \underline{\hspace{2cm}}$

5.  $((7^{-3})^{-1})^{-2} = \underline{\hspace{2cm}}$

10.  $\left(\frac{(2^{-4} \cdot 3^{-2})(4^2 \cdot 9^{-4})}{3^{-10} \cdot 2^0}\right) = \underline{\hspace{2cm}}$

D. Simplify and express the results without zero or negative exponents.

1.  $\frac{5^{-1} x^{-2} y^3 z^0}{5^{-2} x^{-3} y^{-3} z^{-1}}$

9.  $\frac{y^{-6} - x^{-6}}{(x^2 - y^2)x^{-6}y^{-6}}$

2.  $\frac{m^{-4-2}}{m^2}$

10.  $\frac{x^{-1}y^{-2} + 3x^{-2}y^{-1}}{y^{-2} - 9x^{-2}}$

3.  $\left(\frac{x^0}{y^2}\right)^{-1}$

11.  $-3(x+2)(x-2)^{-2} + (x-2)^{-1}$

4.  $\frac{a^{-1} + b^{-1}}{a^{-1} - b^{-1}}$   
 $4)^{-1}$

12.  $2(2x-3)^{-2}(3x+4)^{-2} + 3(2x-3)^{-3}(3x+4)^{-1}$

$$5. \frac{a^{-2} - b^{-2}}{a^{-2}b^{-2}}$$

$$13. 2(x + y)(x^2 - y^2)^{-1} + (x^2 + xy + y^2)(x^3 - y^3)^{-1}$$

$$6. \frac{x^{-2}y^{-1} + x^{-1}y^{-2}}{x^{-1}y^{-1}}$$

$$14. \frac{a^{-2} - a^{-3}y^{-2}}{a^{-3}y^{-2} - y^{-2}}$$

$$7. \left( \frac{c^{-5}e^{-4}l^2}{l^{-1}a^{-1}c^{-6}e^{-5}l^{-1}} \right)^{-1}$$

$$15. (x^0 - 2)(x^0 + 2)$$

$$8. \left( \left( \frac{v^{-1}w^0}{w^2v^{-2}} \right)^{-1} \right)^{-2}$$

**E. Simplify.**

$$1. x^{2a}x^4$$

$$5. (x^a y^{a-3})^3$$

$$2. x^{3a}x^{2b}$$

$$6. (c^3d)^a (cd^7)^a$$

$$3. x^5x^{2a-4}$$

$$7. x^2(x^{a+2}y^3)$$

$$4. (a^{n+1}b^{m+2})^3$$

**F. Answer the following:**

1. Find the volume of a cube with sides of length  $3x$ .
2. Suppose the cube has sides of length  $5x$ . What would its volume be?
3. Suppose the cube has sides of length  $9x$ . What would its volume be?
4. How do the volumes of each of the cubes above change as the dimension doubles? Triples?

<b>Lesson 1.2</b>	<b>Concept of Roots</b>
	<b>OBJECTIVES:</b> 1. To define real and non real roots of a radical. 2. To find the real roots of a radical.

We have learned that  $2^5 = 32$ . In this expression, 2 is called the **base**, 5 is the **exponent** and 32, the **exponential value or product**.

Consider the following examples.

Evaluate.

- |    |           |            |
|----|-----------|------------|
| 1. | $3^3$     | $(-3)^3$   |
| 2. | $3^4$     | $(-3)^5$   |
| 3. | $(0.2)^2$ | $(-0.2)^2$ |
| 4. | $5^4$     | $(-5)^2$   |
| 5. | $2^3$     | $(-2)^3$   |
| 6. | $5^3$     | $(-5)^3$   |

The examples illustrate the following:

- (1) Even power of both positive and negative numbers is positive.
- (2) Odd power of a positive number is positive but the odd power of a negative number is negative.

In general, we can say that if  $n$  is even and

(i) if  $a$  is a negative number, then there is no real  $n$ th root

(ii) if  $a$  is a positive number, then there are two numbers,  $b > 0$  and  $b < 0$ , such that,

$$b^n = a.$$

And, we say that  $a$  is the  $n$ th root of  $b$ .

In this case,  $b > 0$  ( $+b$ ) and  $b < 0$  ( $-b$ ) are both  $n$ th roots of  $a$  and we write in symbols,

$$\sqrt[n]{a} = \pm b.$$

If written separately, we write

$\sqrt[n]{a} = +b$  as the principal  $n$ th root

$-\sqrt[n]{a} = -b$  as the negative  $n$ th root

If  $n$  is odd, then for any real number  $a$ , positive or negative, there is exactly one number  $b$  such that

$$b^n = a \text{ and } \sqrt[n]{a} = b.$$

If  $a$  is positive, the  $n$ th root of  $b$  is positive. If  $a$  is negative, the  $n$ th root of  $b$  is negative.

In  $\sqrt[n]{a} = b$ , the notation  $\sqrt{\phantom{x}}$  is called a **radical sign**,  $n \geq 2$  is called the **index** and  $a$  the **radicand**. Any expression with radical sign is called **radical**.

In  $\sqrt[5]{32}$ , what is the index and what is the radicand?

Let's simplify by getting the  $n$ th root or the value of the following radicals.

1.  $\sqrt{169}$

6.  $\sqrt[4]{81}$

2.  $\sqrt{225}$

7.  $\sqrt[6]{-64}$

3.  $\sqrt[5]{32}$

8.  $\sqrt[3]{0.008}$

4.  $\sqrt[3]{-125}$

5.  $\sqrt{\frac{9}{16}}$

What do you notice about the roots? How do you describe all of them in general?

If the roots or values are exact or rational, how do you describe the given radicands?

Can you give another perfect  $n$ th root in which the root is rational?

Do all  $n$ th roots rational?

***If the radicand is not a perfect  $n$ th root, the  $n$ th root is irrational.***

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**WORKSHEET 2**

**A.** Find the roots of the following.

1.  $\sqrt{81}$

6.  $-\sqrt{\frac{64}{121}}$

2.  $\sqrt{196}$

7.  $\sqrt{\frac{36}{169}}$

3.  $\sqrt{225}$

8.  $\sqrt{\frac{4}{49}}$

4.  $-\sqrt{400}$

9.  $-\sqrt{0.01}$

5.  $\sqrt{\frac{49}{9}}$

10.  $\sqrt{1.44}$

**B.** Find the indicated roots of the following.

1.  $\sqrt[3]{-8}$

7.  $\sqrt[4]{\frac{1}{16}}$

2.  $\sqrt[4]{81}$

8.  $\sqrt[4]{\frac{16}{81}}$

3.  $\sqrt[5]{-32}$

9.  $\sqrt[6]{2^{54}}$

4.  $\sqrt[6]{64}$

10.  $\sqrt[9]{4^{27}}$

5.  $\sqrt[3]{\frac{8}{27}}$

11.  $\sqrt[8]{5^{64}}$

6.  $\sqrt[3]{\frac{-27}{8}}$

12.  $\sqrt[3]{10^{39}}$

C. Perform the indicated operations.

1.  $\sqrt{36} - \sqrt{9} + \sqrt{100}$

2.  $\sqrt{225} - \sqrt{121} + \sqrt{81}$

3.  $2\sqrt{49} + 5\sqrt{25}$

4.  $\sqrt{0.0036} - \sqrt{0.09}$

5.  $\sqrt{64} + \sqrt{\frac{25}{16}}$

6.  $\sqrt{\frac{225}{9}} + \sqrt{\frac{9}{4}} - \sqrt{\frac{128}{2}}$

7.  $\sqrt{\frac{16}{49}} - \sqrt{\frac{36}{25}} + \sqrt{\frac{144}{225}}$

8.  $\sqrt{0.25} + \sqrt{0.0225} + \sqrt{0.81}$

<b>Lesson 1.3</b>	<b>Converting Rational Exponents to Radicals</b>
	<b>OBJECTIVE:</b> To convert algebraic expressions raised to a rational exponent to radicals.

The definition of exponent and the laws are beyond extended integers so that any rational number can be used as an exponent.

Let's consider this particular example

Since  $2^3 = 8$  and  $\left(8^{1/3}\right)^3 = 8$ ,  
 $2^3 = \left(8^{1/3}\right)^3$ , therefore  
 $2 = 8^{1/3}$

We can now say that since the cube of 2 is  $\left(8^{1/3}\right)^3$ , then **2 is the cube root of 8.**

$$2 = 8^{1/3} \quad \text{or} \quad 2 = \sqrt[3]{8}$$

In general,  
 For every real number  $a$ ,  $\left(a^{1/n}\right)^n = a$  where  $n$  is a positive integer.  
 In other words,  $a^{1/n}$  (read  $a$  to the exponent  $\frac{1}{n}$ ) is the principal  $n$ th root of  $a$ .

What number does each of these expressions stand for?

1.  $32^{1/5}$
2.  $27^{1/3}$
3.  $125^{1/3}$
4.  $(-32)^{1/5}$

5.  $(-27)^{1/3}$

6.  $(-100,000)^{1/5}$

Using the laws of exponents for raising to a power and our definition for  $a^{1/n}$ .  
What number will each of these expressions stand for?

$$\left(81^{1/2}\right)^3 = \left(81^{3/2}\right) = \left(81^3\right)^{1/2} = \left((9^2)^3\right)^{1/2} = \left(9^6\right)^{1/2} = \left((3^2)^6\right)^{1/2} = \left(3^{12}\right)^{1/2} = 3^{12/2} = 3^6 = 729$$

$$\left(8^{1/3}\right)^2 = \left(8^2\right)^{1/3} = \left((2^3)^2\right)^{1/3} = \left(2^6\right)^{1/3} = 2^2 = 4$$

$$\left(1000^{1/3}\right)^4 = \left(1000^4\right)^{1/3} = \left((10^3)^4\right)^{1/3} = \left(10^{12}\right)^{1/3} = 10^4 = 10,000$$

The examples above show a way of interpreting such expressions as

$$81^{2/3}$$

$$8^{2/3}$$

$$1000^{4/3}$$

Take  $8^{2/3}$

$8^{2/3}$  may stand for  $(8^2)^{1/3}$  or  $64^{1/3}$  or 4.

$8^{2/3}$  may also stand for  $(8^{1/3})^2$  or  $2^2$  which is also 4.

For every pair of integers  $m$  and  $n$ ,  $n \geq 2$  and for every real number  $a$  for which

$(a^{1/n})^m$  is real

$$a^{m/n} = \left(a^{1/n}\right)^m \quad \text{or} \quad \left(a^m\right)^{1/n}$$

What number does each of these expressions stand for?

1.  $25^{3/2}$

2.  $16^{2/4}$

3.  $100^{5/2}$

4.  $125^{4/3}$

5.  $64^{2/3}$

6.  $(-27)^{5/3}$

If we are going to change the expressions like  $32^{1/5} = 2$ ,  $27^{1/3} = 3$  and  $25^{3/2} = 125$  with rational exponents into a radical expression, we would have

$$32^{1/5} = 2 \quad \longrightarrow \quad \sqrt[5]{32} = 2$$

$$27^{1/3} = 3 \quad \longrightarrow \quad \sqrt[3]{27} = 3$$

$$25^{3/2} = 125 \quad \longrightarrow \quad \sqrt{25^3} = 125$$

In general,

$$a^{1/n} = \sqrt[n]{a} \quad \text{and} \quad a^{m/n} = \sqrt[n]{a^m} \quad \text{or} \quad a^{m/n} = (\sqrt[n]{a})^m$$

for integers  $m$  and  $n$ ,  $n \geq 2$  and for real number  $a$  where  $\sqrt[n]{a}$  is real. Since

$$\left(a^{1/n}\right)^n = a, \text{ it is also true that } \left(\sqrt[n]{a}\right)^n = a.$$

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**WORKSHEET 3**

*A. What is another way of writing each of the following:*

1.  $81^{1/2}$

4.  $\sqrt[5]{-243}$

2.  $64^{3/2}$

5.  $\sqrt[3]{0.008}$

3.  $36^{1/2}$

*B. What number does each expression represent?*

1.  $(\sqrt[5]{2})^2$

2.  $(\sqrt[5]{-2})^5$

3.  $(\sqrt[4]{5})^4$

4.  $(\sqrt[4]{-5})^4$

*C. In which expression will  $\sqrt{a}$  not be a real number?*

1.  $\sqrt[3]{-64}$

4.  $\sqrt[6]{-64}$

2.  $\sqrt[4]{-64}$

5.  $(\sqrt{1})^2$

3.  $\sqrt[6]{64}$

6.  $(\sqrt{-1})^2$



### **Academic Workload for Trigonometry 3 AS**

The first topic in trigonometry is review of the Pythagorean Theorem. Last year, we discussed and proved this theorem using similarity. We note that there are a lot of proofs for the Pythagorean theorem, some are simple and some are very complicated.

The first part of the workload is to research on proofs of the Pythagorean theorem other than the similarity proof. You are to study only one of the many proofs and explain thoroughly using your own words. This chosen proof should be typewritten with accompanying diagrams.

The second part of the workload is simply some problems to solve using the Pythagorean theorem. You may write your answers on intermediate pad and attach with part 1.

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**Trigonometry**

**Name:** \_\_\_\_\_

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**Solve the following problems. Copy and Answer.**

A. Write R if the given numbers are sides of a right triangle and N if they are not.

1. 3, 4, 5
2. 6, 8, 10
3. 10, 12, 15
4. 7, 20, 23
5. 10, 20, 30

B. Find the missing side of the following triangles with "c" as the hypotenuse.

1.  $a = 7, b = 24, c = \underline{\hspace{2cm}}$
2.  $b = 15, c = 17, a = \underline{\hspace{2cm}}$
3.  $a = 9, c = 41, b = \underline{\hspace{2cm}}$
4.  $a = 20, c = 21, b = \underline{\hspace{2cm}}$
5.  $b = 35, c = 37, a = \underline{\hspace{2cm}}$

C. Solve the following completely

1. Village A is 9 miles from Village B, and 12 miles from Village C. A road connects Village B and C directly. Find the length of this road.
  
  
  
  
  
  
  
  
  
  
2. Rene wants to swim across a river that is 400 meters wide. He begins swimming perpendicular to the shore he started from but ends up

100 meters down river from where he started because of the current. How far did he actually swim from his starting point?

3. Dennis placed a 13 feet ladder 5 feet away from a wall. The distance from the ground straight up to the top of the wall is 13 feet. Will the ladder reach the top of the wall?

4. Marvin bought an LCD monitor for his computer and the sales man offers a 19-inch monitor. We know that this is the diagonal distance across the screen. If the screen measures 10 inches in height, what is the actual width of the screen to the nearest inch?

5. Darius is standing at the first base of a baseball diamond, which is a square with sides of 90 feet. What is the shortest distance, to the nearest tenth of a foot, from where he stands up to the third base?